

Describe Proportional Relationships: Constant of Proportionality

Let's Build

Lesson 2-4

I can ... use the constant of proportionality in an equation to represent a proportional relationship.

Explore and Share

Jamal can run 1 mile in 5.05 minutes. If Jamal maintains this pace during a 5-kilometer (5K) race, he expects to break the course record of 15.25 minutes. Is Jamal's expectation reasonable? Explain.



Be Precise How can you convert 5 kilometers to miles?

Reasoning Assuming that Jamal runs at a constant rate, how does his pace describe the time it takes him to finish a race of any length?

Essential Question

How can you represent a proportional relationship with an equation?

Animation

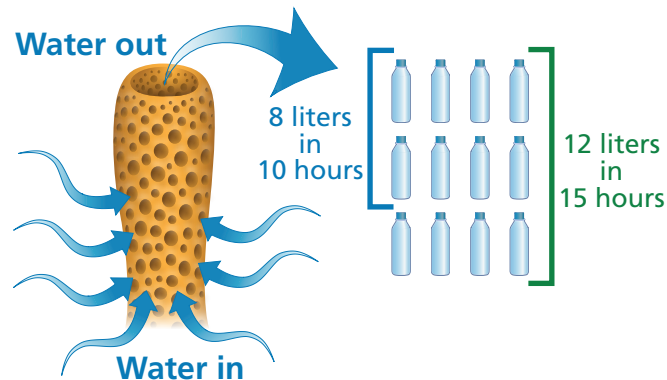


Visual Learning

Example 1 Write an Equation to Represent a Proportional Relationship

A sponge is an example of a *filter feeder*. It takes in food by filtering water through its body. The sponge maintains a constant flow of water through its body.

What is an equation that represents the proportional relationship between the time and the amount of water filtered?



The **constant of proportionality** is the constant multiple that relates proportional quantities x and y . It is the value of the ratio $\frac{y}{x}$ and is represented by k .

Make a table to find the constant of proportionality.

Hours (x)	Liters (y)	$\frac{\text{Liters}}{\text{Hours}} \left(\frac{y}{x}\right)$
10	8	$\frac{8}{10} = 0.8$
15	12	$\frac{12}{15} = 0.8$

The constant of proportionality, k , is 0.8.

You can use k to write an equation that represents a proportional relationship.

$$k = \frac{y}{x}$$

$$kx = y$$

$$y = kx$$

You can use an equation in the form $y = kx$ to represent any proportional relationship.

Use the equation to represent the relationship between time and the amount of water filtered.

$$y = 0.8x$$

Try It! Maria made two batches of fruit punch. The table at the right shows how many quarts of juice she used for each batch. Write an equation that relates the proportional quantities.

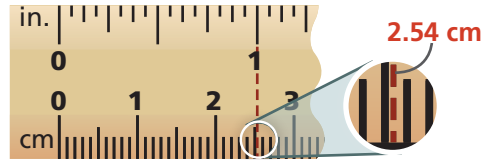
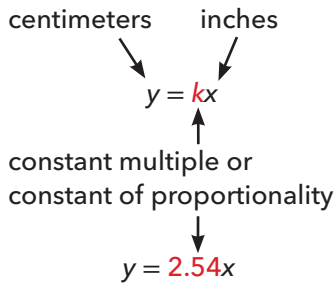
Apple Juice (x)	Grape Juice (y)	$\frac{\text{Grape Juice}}{\text{Apple Juice}} \left(\frac{y}{x}\right)$
5	8	
10	16	

Example 2 Solve Problems Using an Equation

1 inch = 2.54 centimeters. How many centimeters long is an 18-inch ruler?

STEP 1

Write an equation to represent this relationship.



STEP 2

Use the equation to find the length of the ruler in centimeters.

$$y = 2.54x$$

$$y = 2.54(18)$$

$$y = 45.72$$

The ruler is 45.72 centimeters long.

Try It! A florist sells a dozen roses for \$35.40. She sells individual roses for the same unit cost. Write an equation to represent the relationship between the number of roses, x , and the total cost of the roses, y . How much would 18 roses cost?

Example 3 Determine Whether $y = kx$ Describes a Situation

Can you represent the total cost, y , for admission and x rides at the amusement park using an equation in the form $y = kx$? Explain.

STEP 1

Make a table that shows the total cost.

Rides	Cost
1	\$25
2	\$30
3	\$35

STEP 2

Compare the ratios to determine whether the relationship is proportional.

$$\frac{\$25}{1} = \$25$$

$$\frac{\$30}{2} = \$15$$

$$\frac{\$35}{3} = \$11.67$$

This relationship is not proportional, so it cannot be represented with an equation of the form $y = kx$.



Try It! Balloon A is released 5 feet above the ground. Balloon B is released at ground level. Both balloons rise at a constant rate.

Which situation can you represent using an equation of the form $y = kx$? Explain.

Balloon A

Time (s)	Height (ft)
1	19
2	13
3	17

Balloon B

Time (s)	Height (ft)
1	4
2	8
3	12

Key Concept

Two proportional quantities x and y are related by a constant multiple, or the constant of proportionality, k .

You can represent a proportional relationship using the equation $y = kx$.

Talk About Math Ideas

How is a constant of proportionality like a unit rate?

Do You Understand?

- 1. Essential Question** How can you represent a proportional relationship with an equation?
- 2. Generalize** How can you use an equation to find an unknown value in a proportional relationship?
- 3. Reasoning** Why does the equation $y = 3x + 5$ NOT represent a proportional relationship?

Do You Know How?

- 4.** Determine whether each equation represents a proportional relationship. If it does, identify the constant of proportionality.
 - a.** $y = 0.5x - 2$
 - b.** $y = 1,000x$
 - c.** $y = x + 1$
- 5.** The manager of a local park estimates that she needs 3 trash bags for every 5 people who attend a park cleanup. If 1,200 people volunteer for the park cleanup, how many trash bags should the manager order?
- 6.** A half dozen water bottle filters cost \$15. What constant of proportionality relates the number of water bottle filters and total cost? Write an equation that represents this relationship.



Practice and Problem Solving

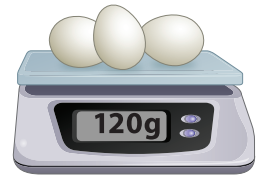
7. What is the constant of proportionality in the equation $y = 5x$?

8. What is the constant of proportionality in the equation $y = 0.41x$?

9. The equation $P = 3s$ represents the perimeter P of an equilateral triangle with side length s . Is there a proportional relationship between the perimeter and the side length of an equilateral triangle? Explain.

10. **Model with Math** In a chemical compound, there are 3 parts zinc for every 16 parts copper, by mass. A piece of the compound contains 320 grams of copper. Write and solve an equation to determine the amount of zinc in the chemical compound.

11. The weight of 3 eggs is shown. Assuming the three eggs are all the same weight, find the constant of proportionality.



12. The height of a stack of video game cases is proportional to the number of cases in the stack. The height of 6 video game cases is 114 mm.

- a. Write an equation that relates the height, y , of a stack of video game cases and the number of cases, x , in the stack.
- b. What would be the height of 13 video game cases?

13. Ann's car can travel 228 miles on 6 gallons of gas.

- a. Write an equation to represent the distance, y , in miles Ann's car can travel on x gallons of gas.
- b. Ann's car used 7 gallons of gas during a trip. How far did Ann drive?

14. The value of a baseball player's rookie card began to increase once the player retired in 1996. The value has increased by \$2.52 each year since then.

a. How much was the baseball card worth in 1997? In 1998? In 1999?

b. **Construct Arguments** Why is there not a proportional relationship between the years since the player retired and the card value? Explain.



15. **Higher Order Thinking** A car travels $2\frac{1}{3}$ miles in $3\frac{1}{2}$ minutes at a constant speed.

a. Write an equation to represent the distance the car travels, d , in miles for m minutes.

b. Write an equation to represent the distance the car travels, d , in miles for h hours.

Assessment Practice

16. For every ten sheets of stickers you buy at a craft store, the total cost increases \$20.50.

Write an equation that relates the number of sheets purchased, x , and the total cost, y , of the stickers.

Cost of Stickers

Number of Sheets (x)	3			19
Total Cost (y)		\$10.25	\$26.65	

Use the equation you wrote to complete the table.

17. 600,000 gallons of water pass through a given point along a river every minute. Which equation represents the amount of water, y , that passes through the point in x minutes?

Ⓐ $x = 10,000y$

Ⓑ $y = 600,000x$

Ⓒ $y = 10,000x$

Ⓓ $y = 600,000 + x$